**1. ABCDE is a regular pentagon with F at its center. How many different triangles can be formed by joining 3 of the points A,B,C,D,E and F?**
(A) 10
(B) 15
(C) 20
(D) 25
(E) 30

Answer: C.

**2. The function f is defined for all positive integers n by the following rule: f(n) is the number of positive integers each of which is less than n and has no positive factor in common with n other than 1. If p is prime, then f(p) =**
(A) P-1
(B) P-2
(C) (P+1)/2
(D) (P-1)/2
(E) 2

Answer: A.

**3. How many numbers that are not divisible by 6 divide evenly into 264,600?**
(A) 9
(B) 36
(C) 51
(D) 63
(E) 72

Answer: D.

**4.A certain quantity is measured on two different scales, the R-scale and the S-scale, that are related linearly. Measurements on the R-scale of 6 and 24 correspond to measurements on the S-scale of 30 and 60, respectively. What measurement on the R-scale corresponds to a measurement of 100 on the S-scale?**
(A) 20
(B) 36
(C) 48
(D) 60
(E) 84

Answer: C.

**5. Mrs. Smith has been given film vouchers. Each voucher allows the holder to see a film without charge. She decides to distribute them among her four nephews so that each nephew gets at least two vouchers. How many vouchers has Mrs. Smith been given if there are 120 ways that she could distribute the vouchers?**
(A) 13
(B) 14
(C) 15
(D) 16
(E) more than 16

Answer: C.

**6. This year Henry will save a certain amount of his income, and he will spend the rest. Next year Henry will have no income, but for each dollar that he saves this year, he will have 1 + r dollars available to spend. In terms of r, what fraction of his income should Henry save this year so that next year the amount he was available to spend will be equal to half the amount that he spends this year?**
(A) 1/(r+2)
(B) 1/(2r+2)
(C) 1/(3r+2)
(D) 1/(r+3)
(E) 1/(2r+3)

Answer: E.

**7. Before being simplified, the instructions for computing income tax in Country Rwere to add 2 percent of one's annual income to the average(arithmetic mean)of 100units of Country R's currency and 1 percent of one's annual income. Which of the following represents the simplified formula for computing the income tax in Country R's currency, for a person in that country whose annual income is I?**
(A) 50+I/200
(B) 50+3I/100
(C) 50+I/40
(D) 100+I/50
(E) 100+3I/100

Answer: C.

**8. How many positive integers less than 10,000 are such that the product of their digits is 210?**
(A) 24
(B) 30
(C) 48
(D) 54
(E) 72

Answer: D.

**9. Find the number of selections that can be made taking 4 letters from the word"ENTRANCE".**
(A) 70
(B) 36
(C) 35
(D) 72
(E) 32

Answer:B.

Find in the above word, the number of arrangements using the 4 letters.

**10. How many triangles with positive area can be drawn on the coordinate plane such that the vertices have integer coordinates (x,y) satisfying 1≤x≤3 and 1≤y≤3?**
(A) 72
(B) 76
(C) 78
(D) 80
(E) 84

Answer: B.

**1. ABCDE is a regular pentagon with F at its center. How many different triangles can be formed by joining 3 of the points A,B,C,D,E and F?**
(A) 10
(B) 15
(C) 20
(D) 25
(E) 30

Regular pentagon is a pentagon where all sides are equal. In such pentagon center is not collinear to any two vertices, so ANY three points (from 5 vertices and center point) WILL form the triangle.

The question basically asks how many triangles can be formed from the six points on a plane with no three points being collinear.

As any 3 points from 6 will make a triangle (since no 3 points are collinear), then:

6C3=20

Answer: C.

Hope it helps.

**2.The function f is defined for all positive integers n by the following rule. f(n) is the number of positive integers each of which is less than n and has no positive factor in common with n other than 1. If p is a prime number then f(p)=**
A. p-1
B. p-2
C. (p+1)/2
D. (p-1)/2
E. 2

If not the wording the question wouldn't be as tough as it is now. The GMAT often hides some simple concept in complicated way of delivering it.

This question for instance basically asks: how many positive integers are less than given prime number p which have no common factor with p except 1.

Well as p is a prime, all positive numbers less than p have no common factors with p (except common factor 1). So there would be p-1 such numbers (as we are looking number of integers less than p).

For example: if p=7 how many numbers are less than 7 having no common factors with 7: 1, 2, 3, 4, 5, 6 --> 7-1=6.

Answer: A.

Hope it's clear.

**3. How many numbers that are not divisible by 6 divide evenly into 264,600?**
(A) 9
(B) 36
(C) 51
(D) 63
(E) 72

Answer: D.

First of all you should know the formula counting the number of distinc factors of an integer:

You have to write the number as the product of primes as **a^p*b^q*c^r**, where a, b, and c are prime factors and p,q, and r are their powers.

**The number of factors the number contains will be expressed by the formula (p+1)(q+1)(r+1).**
Let's take an example for clear understanding:Find the number of all (distinct) factors of 1435:
1. 1435 can be expressed as 5^1*17^1*19^1
2. total number of factors of 1435 including 1 and 1435 itself is (1+1)*(1+1)*(1+1)=2*2*2=8 factors.

OR
Distinct factors of 18=2*3^2 --> (1+1)*(2+1)=6. Lets check: factors of 18 are: 1, 2, 3, 6, 9 an 18 itself. Total 6.

Back to our question:
How many numbers that are not divisible by 6 divide evenly into 264,600?

264,600=2^3*3^3*5^2*7^2

We should find the factor which contain no 2 and 3 together, so not to be divisible by 6.

Clearly, the factors which contain only 2,5,7 and 3,5,7 won't be divisible by 6. So how many such factors are there?
2^3*5^2*7^2 --> (3+1)*(2+1)*(2+1)=36 (the product of powers of 2, 5,and 7 added 1)

3^3*5^2*7^2 --> (3+1)*(2+1)*(2+1)=36 (the product of powers of 2, 5, and 7 added 1)

So 36+36=72. BUT this number contains duplicates:

For example: 2^3*5^2*7^2--> (3+1)*(2+1)*(2+1)=36 This 36 contains the factors when the power of 2 is 0 (2^0=1)--> 2^0*5^2*7^2 giving us only the factors which contain 5-s and/or 7-s. (5*7=35, 5*7^2=245, 5^2*7=175, 5*7^0=5, 5^0*7=7....) number of such factors are (2+1)*(2+1)=9 (the product of powers of 5 and 7 added 1).

And the same factors are counted in formula 3^3*5^2*7^2 --> (3+1)*(2+1)*(2+1)=36: when power of 3 is 0 (3^0=1). --> 5*7=35, 5*7^2=245, 5^2*7=175, 5*7^0=5, 5^0*7=7.... such factors are (2+1)*(2+1)=9. (the product of powers of 5 and 7 added 1).

So we should subtract this 9 duplicated factors from 72 --> 72-9=63. Is the correct answer.

The problem can be solved from another side:
264,600=2^3*3^3*5^2*7^2 # of factors= (3+1)(3+1)(2+1)(2+1)=144. So our number contains 144 distinct factors. # of factors which contain 2 and 3 is 3*3=9 (2*3, 2^2*3, 2^3*3, 2*3^2, 2^2*3^2, 2^3*3^2, 2*3^3, 2^2*3^3, 2^3*3^3 total 9) multiplied by (2+1)*(2+1)=9 (powers of 5 and 7 plus 1) --> 9*9=81 ---> 144-81=63.

Hope now it's clear.

**4.A certain quantity is measured on two different scales, the R-scale and the S-scale, that are related linearly. Measurements on the R-scale of 6 and 24 correspond to measurements on the S-scale of 30 and 60, respectively. What measurement on the R-scale corresponds to a measurement of 100 on the S-scale?**
(A) 20
(B) 36
(C) 48
(D) 60
(E) 84

Solution:
this is like linear equation problem
where r =sa +b --(1) or y = mx + c
put r=6 then s=30
and r= 24 then s =60
thus 2 equations
6 =30a+b-------------(A)
24=60a+b------------(B)
solve (A) and (B)
thus we get
a =18/30 and c =-12
thus if s =100 then r = (18/30)*100-12 =48

Answer is C or (3) here.

**5. Mrs. Smith has been given film vouchers. Each voucher allows the holder to see a film without charge. She decides to distribute them among her four nephews so that each nephew gets at least two vouchers. How many vouchers has Mrs. Smith been given if there are 120 ways that she could distribute the vouchers?**
(A) 13
(B) 14
(C) 15
(D) 16
(E) more than 16

Answer: C.

Clearly there are more than 8 vouchers as each of four can get at least 2. So, basically 120 ways vouchers can the distributed are the ways to distribute $x-8$vouchers, so that each can get from zero to $x-8$ as at "least 2", or 2*4=8, we already booked. Let $x-8$ be $k$.

In how many ways we can distribute $k$ identical things among 4 persons? Well there is a formula for this but it's better to understand the concept.

Let $k=5$. And imagine we want to distribute 5 vouchers among 4 persons and each can get from zero to 5, (no restrictions).

Consider:

$ttttt|||$
We have 5 tickets (t) and 3 separators between them, to indicate who will get the tickets:

$ttttt|||$
Means that first nephew will get all the tickets,

$|t|ttt|t$
Means that first got 0, second 1, third 3, and fourth 1

And so on.

How many permutations (arrangements) of these symbols are possible? Total of 8 symbols (5+3=8), out of which 5 $t$'s and 3 $|$'s are identical, so $\frac{8!}{5!3!}=56$. Basically it's the number of ways we can pick 3 separators out of 5+3=8: $8C3$.

So, # of ways to distribute 5 tickets among 4 people is $(5+4-1)C(4-1)=8C3$.

For $k$ it will be the same: # of ways to distribute $k$ tickets among 4 persons (so that each can get from zero to $k$ ) would be $(K+4-1)C(4-1)=(k+3)C3=\frac{(k+3)!}{k!3!}=120$.

$(k+1)(k+2)(k+3)=3!*120=720$. --> $k=7$. Plus the 8 tickets we booked earlier: $x=k+8=7+8=15$.

Answer: C (15).

P.S. Direct formula:

**The total number of ways of dividing n identical items among r persons, each one of whom, can receive 0,1,2 or more items is** $n+r-1C_{r-1}$.

**The total number of ways of dividing n identical items among r persons, each one of whom receives at least one item is** $n-1C_{r-1}$.

Hope it helps.

**6. This year Henry will save a certain amount of his income, and he will spend the rest. Next year Henry will have no income, but for each dollar that he saves this year, he will have 1 + r dollars available to spend. In terms of r, what fraction of his income should Henry save this year so that next year the amount he was available to spend will be equal to half the amount that he spends this year?**
(A) 1/(r+2)
(B) 1/(2r+2)
(C) 1/(3r+2)
(D) 1/(r+3)
(E) 1/(2r+3)

SOLUTION:
$x$ fraction of saving, $I$ income.

$(1-x)*I=2*x*I*(1+r)$, $I$ cancels out.

$x=\frac{1}{3+2r}$

Answe: E.

**7. Before being simplified, the instructions for computing income tax in Country Rwere to add 2 percent of one's annual income to the average(arithmetic mean)of 100units of Country R's currency and 1 percent of one's annual income. Which of the following represents the simplified formula for computing the income tax in Country R's currency, for a person in that country whose annual income is I?**
(A) 50+I/200
(B) 50+3I/100
(C) 50+I/40
(D) 100+I/50
(E) 100+3I/100

Tax is the sum of the following:
2 percent of one's annual income - $0.02I$;
The average (arithmetic mean) of 100 units of country R's currency and 1 percent of one's annual income - $\frac{100+0.01I}{2}$.

$$Tax=0.02*I+\frac{100+0.01*I}{2}=\frac{0.04*I+100+0.01*I}{2}=50+\frac{0.05*I}{2}=50+\frac{I}{40}.$$

Answer: C.

**8. How many positive integers less than 10,000 are such that the product of their digits is 210?**
(A) 24
(B) 30
(C) 48
(D) 54
(E) 72

210=1*2*3*5*7=1*6*5*7. (Only 2*3 makes the single digit 6).

So, four digit numbers with combinations of the digits {1,6,5,7} and {2,3,5,7} and three digit numbers with combinations of digits {6,5,7} will have the product of their digits equal to 210.

{1,6,5,7} # of combinations 4!=24

{2,3,5,7} # of combinations 4!=24
{6,5,7} # of combinations 3!=6

24+24+6=54.

Answer: D.

**9. Find the number of selections that can be made taking 4 letters from the word "ENTRANCE".**
(A) 70
(B) 36
(C) 35
(D) 72
(E) 32

This problem is not the one you'll see on real GMAT. As combinatorics problems on GMAT are quite straightforward. But still it could be good for practice.

We have 8 letters from which 6 are unique.

Possible scenarios for 4 letter selection are:
A. All letters are different;
B. 2 N-s and other letters are different;
C. 2 E-s and other letters are different;
D. 2 N-s, 2 E-s.

Let's count them separately:
A. All letters are different, means that basically we are choosing 4 letters form 6 distinct letters: 6C4=15;
B. 2 N-s and other letters are different: 2C2(2 N-s out of 2)*5C2(other 2 letters from distinct 5 letters left)=10;
C. 2 E-s and other letters are different: 2C2(2 E-s out of 2)*5C2(other 2 letters from distinct 5 letters left)=10;;
D. 2 N-s, 2 E-s: 2C2*2C2=1.

15+10+10+1=36

Answer: B.

**Finding in the above word, the number of arrangements using the 4 letters.**

This one should go relatively easy after we solved the previous. So we have:

A. 15 4 letter words with all distinct letters. # of arrangements of 4 letter word is 4!, as we have 15 such words, then = 15*4!=360;
B. 10 4 letter words with two N-s and two other distinct letters. The same here except # of arrangements would be not 4! but 4!/2! as we need factorial correction to get rid of the duplications=10*4!/2!=120;
C. The same as above: 10*4!/2!=120;
D. 2 N-s and 2 E-s, # of arrangement=4!/2!*2!=6.

Total=360+120+120+6=606.

Hope it's clear.

**10. How many triangles with positive area can be drawn on the coordinate plane such that the vertices have integer coordinates (x,y) satisfying 1≤x≤3 and 1≤y≤3?**
(A) 72
(B) 76
(C) 78
(D) 80
(E) 84

It would be better if you draw it while reading this explanation. With the restriction given (1≤x≤3 and 1≤y≤3) we get 9 points, from which we can form the triangle: (1,1), (1,2), (1,3), (2,1)...

From this 9 points any three (9C3) will form the triangle BUT THE SETS of three points which are collinear.

We'll have 8 sets of collinear points of three:

3 horizontal {(1,1),(2,1),(3,1)} {(1,2)(2,2)(3,2)}...
3 vertical
2 diagonal {(1,1)(2,2)(3,3)}{(1,3)(2,2)(3,1)}

So the final answer would be; 9C3-8=84-8=76

Answer: B.

Hope it's clear.